Math 21C-2
Practice Midterm

Name: __________________________________________

Signature: ______________________________________

Student ID: _________________________________

• There are ten (plus cover and bonus) pages to the exam.
• The exam totals 100 points, plus 10 bonus points.
• You will have 90 minutes to complete the exam.
• No calculators, notes, or books allowed.
• Good luck!

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1. (10 points) Definitions and Examples:

   a. (2 points) Write the definition for a sequence to be bounded above. (An example is **not** sufficient for full credit.)

   b. (2 points) Write the definition of an alternating series. (An example is **not** sufficient for full credit.)

   c. (2 points) Write the definition for a sequence to diverge to infinity. (An example is **not** sufficient for full credit.)

   d. (2 points) Write the definition of the cross product of two vectors. (An example is **not** sufficient for full credit.)

   e. (2 points) Let $f = f(x)$. Write the definition of the Maclaurin series for $f$. (An example is **not** sufficient for full credit.)
2. (10 points)  Short Answers

a. (5 points)  State the \textit{Integral Test} for infinite series.

b. (5 points)  State the \textit{nth-Term test} for infinite series.
3. (10 points) Determine whether the following sequences converge or diverge. If a sequence converges, find its limit.

a. (3 points) \( a_n = \ln(n + 1) - \ln(n) \).

b. (3 points) \( b_n = n2^{-\ln(n)} \).

c. (4 points) \( a_n = \frac{n!}{(-3)^n} \).
4. (10 points) Determine whether the following series converge conditionally, converge absolutely, or diverge.

a. (5 points) \( \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n} \).

b. (5 points) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{0.999999}} \).
5. (10 points) Determine the values of $x$ for which the following series converges conditionally, converges absolutely, or diverges. What are the center, radius, and interval of convergence?

$$\sum_{n=2}^{\infty} n^n x^n.$$
6. (10 points) Compute (no shortcuts!!) the Taylor series centered at 1 for the function

\[ f(x) = \sqrt{x}. \]
7. (10 points) Using what you know about familiar Taylor series, write a power series for $f(x) = e^{-x^2}$. Use the first 3 non-zero terms to estimate $\int_0^1 e^{-x^2} dx$.

(Hint: you should be familiar with the series for $e^x$.)
8. (10 points) Let \( P \) be the point \((\sqrt{2}/4, \sqrt{2}/4, -1)\), and \( Q \) be the point \((-\sqrt{2}/4, -\sqrt{2}/4, -1)\).

a. (3 points) Find the component form for \( \overrightarrow{PQ} \).

b. (3 points) Find the magnitude \( |\overrightarrow{PQ}| \).

c. (4 points) Find the unit vector in the direction of \( \overrightarrow{PQ} \).
9. (10 points) Let \( \vec{u} = 2\vec{i} + 3\vec{j} - \vec{k} \), and \( \vec{v} = 3\vec{i} - 2\vec{j} + 20\vec{k} \).

a. (3 points) Find \( \vec{u} \cdot \vec{v} \).

b. (3 points) Find \( \vec{u} \times \vec{v} \).

c. (4 points) Find \( \vec{u} \cdot (\vec{u} \times \vec{v}) \).
10. (10 points) Find parametric equations for the line which passes through \((2, 4, 5)\) and is perpendicular to the plane \(3x + 7y - 5z = 21\).
**Bonus.** (10 points) Let $p_n$ denote the $n$th prime: $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$, $p_5 = 11$, etc. Determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{p_n^2}$$